ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
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ON DETERMINATION OF ELASTIC CHARACTERISTICS AND INITIAL STRESSES IN PLATE AND MEMBRANE (INVERSE PROBLEM)

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Սալերում և մեմբրաններում առաձգական բնութագրիչների և նախնական լարումների որոշման վերաբերյալ (հակադարձ խնդիր)

Աշխատանքում փորձ է արվում, օգտագործելով էկսպերիմենտալ արդյունքները, մշակել ռացիոնալ միջոց սալերի և մեմբրանների առաձգական բնութագրիչների և նախնական լարումների որոշման համար:

Որոնվող մեծությունների որոշման համար առաջարկվում է երկու տարբերակ: Առաջին տարբերակ՝ մակերևույթի վրա կիրառված ստատիկ բեռնավորված սալի դեֆորմացված մակերևույթի անալիզի միջոցով: Երկրորդ տարբերակ՝ սալի ազատ տատանման հաճախությունների անալիզի միջոցով:

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К определению упругих характеристик и начальных напряжений в пластинках и мембранах (обратная задача)

В работе делается попытка разработать рациональные способы определения коэффициентов упругости и начальных напряжений в неоднородных пластинках и мембранах с использованием результатов, полученных из экспериментов.

Предлагается два варианта определения искомых величин. Первый вариант-путем анализа деформированной поверхности пластинки под действием статически приложенных поверхностных нагрузок. Второй вариант-путем анализа частот свободных колебаний пластинки.

In the work an attempt is done to constract a rational method for determination of the elastic coefficients and initial stresses in nonhomogeneous plates and membranes by use of experimental results.

Two variants of unknown values determination are suggested. The first variant is the analysis of the plate deflection under action of the surface static forces. The second variant is the analysis of frequencies of the plate free vibration. It is clear, that the combination of two variants for determination of all unknown parameters may be used too.

1.Equations of motion of the considered plate with a constant thickness h, in the cartesian system of coordinates have the form [1].

$$\frac{\partial N_x}{\partial x}+\frac{\partial T_{xy}}{\partial y}=0, \quad \frac{\partial T_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0 \qquad (1.1)$$

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 H}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \left(N_x + N_x^0\right)\frac{\partial^2 w}{\partial x^2} + \left(N_y + N_y^0\right)\frac{\partial^2 w}{\partial y^2} +$$

$$+2\left(T_{xy} + T_{xy}^0\right)\frac{\partial^2 w}{\partial x \partial y} + q(x,y,t) - \rho h \frac{\partial^2 w}{\partial t^2} = 0$$

Here, in contrast to the problem classical statement, the unknown initial planar tensions $N_x^0(x,y), N_y^0(x,y), T_{xy}^0(x,y)$ are included also. In (1.1) we have the known presentations of the plates bending classical nonlinear theory for the moments and tensions

$$N_x = \frac{Eh}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2),\quad N_y = \frac{Eh}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1),\quad T_{xy} = \frac{Eh}{2(1+\nu)}\gamma \tag{1.2}$$

$$M_x = D(\chi_1 + \nu\chi_2),\quad M_y = D(\chi_2 + \nu\chi_1),\quad H = D(1-\nu)\chi_{12} \tag{1.3}$$

$$\varepsilon_1 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2,\quad \varepsilon_2 = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2,\quad \gamma = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}$$

$$\chi_1 = -\frac{\partial^2 w}{\partial x^2},\quad \chi_2 = -\frac{\partial^2 w}{\partial y^2},\quad \chi_{12} = -\frac{\partial^2 w}{\partial x \partial y},\quad D = \frac{Eh^3}{12\left(1-\nu^2\right)} \tag{1.4}$$

In above mentioned in general case it is assumed that unknown values E - Young's moduli, ν-Poisson's ratio, ρ - plate material density and also initial tensions N_x^0, N_y^0, T_{xy}^0 are functions of coordinates x and y.

2.Equations of the plate static bending can be represented as follows, if unknown values are homogeneous [2]

$$D\Delta^2 w = L(w,\Phi) + T_x^0\frac{\partial^2 w}{\partial x^2} + T_y^0\frac{\partial^2 w}{\partial y^2} + 2T_{xy}^0\frac{\partial^2 w}{\partial x \partial y} + q(x,y)$$

$$\Delta^2\Phi = -\frac{Eh}{2}L(w,w),\ L(w,\Phi) \equiv \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 w}{\partial y^2}\frac{\partial^2 \Phi}{\partial x^2} - 2\frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 \Phi}{\partial x \partial y} \tag{2.1}$$

where w is the plate deflection, Φ is the stress function and planar tensions are represented by Φ as follows

$$T_x = \frac{\partial^2\Phi}{\partial y^2},\ T_y = \frac{\partial^2\Phi}{\partial x^2},\ T_{xy} = -\frac{\partial^2\Phi}{\partial x \partial y} \tag{2.2}$$

Let us consider a rectangular plate with the following boundary conditions

$$u = v = w = 0,\ M_x = 0 \text{ when } x = 0, a$$

$$u = v = w = 0,\ M_y = 0 \text{ when } x = 0, b \tag{2.3}$$

The plate deflection is presented in the form

$$W = \sum_{m,n=1} f_{mn}\sin\lambda_m x \sin\mu_n y,\quad \lambda_m = \frac{m\pi}{a},\quad \mu_n = \frac{n\pi}{b} \tag{2.4}$$

Then for nonlinear operator $L(w,w)$ is received [3]

$$L(w,w)=\sum_{m,n=1}^{\infty} C^{(1)}_{mn}\cos\lambda_m x\cos\mu_n y+\sum_{m,n=1}^{\infty} C^{(2)}_{mn}\cos\lambda_m x+\sum_{m,n=1}^{\infty} C^{(3)}_{mn}\cos\mu_n y,$$

where for coefficients $C^{(k)}_{mn}$ we have

$$2C^{(1)}_{mn}=\sum_{\alpha=m+1,\beta=n+1}^{\infty}\lambda_\alpha\mu_{\beta-n}\left(\lambda_\alpha\mu_{\beta-n}-\lambda_{\alpha-m}\mu_\beta\right)f_{\alpha\beta}f_{\alpha-m,\beta-n}+$$

$$+\sum_{\alpha=m+1,\beta=1}^{\infty}\lambda_\alpha\mu_{n+\beta}\left(\lambda_\alpha\mu_{n+\beta}-\lambda_{\alpha-m}\mu_\beta\right)f_{\alpha\beta}f_{\alpha-m,\beta+n}+$$

$$+\sum_{\alpha=1,\beta=n+1}^{\infty}\lambda_\alpha\mu_{\beta-n}\left(\lambda_\alpha\mu_{\beta-n}-\lambda_{m+\alpha}\mu_{\beta-n}\right)f_{\alpha\beta}f_{m+\alpha,\beta-n}+\sum_{\alpha,\beta=1}^{\infty}\lambda_\alpha\mu_{n+\beta}\left(\lambda_\alpha\mu_{n+\beta}-\lambda_{\alpha+m}\mu_{n+\beta}\right)f_{\alpha\beta}f_{m-\alpha,n-\beta}+$$

$$+\sum_{\alpha,\beta=1}^{m-1,n-1}\lambda_\alpha\mu_{n-\beta}\left(\lambda_\alpha\mu_{n-\beta}-\lambda_{m-\alpha}\mu_\beta\right)f_{\alpha\beta}f_{m-\alpha,n-\beta}$$

$$2C^{(2)}_{mn}=\sum_{\alpha=m+1}^{\infty}\lambda_\alpha\mu_n^2(\lambda_\alpha-\lambda_{\alpha-m})f_{\alpha n}f_{\alpha-m,n}+\sum_{\alpha=1}^{\infty}\lambda_\alpha\mu_n^2(\lambda_\alpha-\lambda_{\alpha+m})f_{\alpha n}f_{m+\alpha,n}-\sum_{\alpha=1}^{m-1}\mu_n^2(\lambda_\alpha+\lambda_{m-\alpha})f_{\alpha n}f_{m-\alpha,n}$$

$$2C^{(3)}_{mn}=\sum_{\beta=n+1}^{\infty}\lambda_m^2\mu_{\beta-n}(\mu_{\beta-n}-\mu_\beta)f_{n\beta}f_{m\beta-n}+\sum_{\beta=1}^{\infty}\lambda_m^2\mu_{\beta+n}(\mu_{\beta+n}-\mu_\beta)f_{n\beta}f_{m\beta+n}-\sum_{\beta=1}^{n-1}\lambda_m^2\mu_{n-\beta}(\mu_{n-\beta}-\mu_\beta)f_{n\beta}f_{m,n-\beta}$$

As in brought formulas as also in below, it is necessary to account that members with zero or negative indexes are equal to zero.

In this case the particular solution of the second equation of (2. 1) has the form

$$\Phi=\sum_{m,n=1}^{\infty}\Phi^{(1)}_{mn}\cos\lambda_m x\cos\mu_n y+\sum_{m,n=1}^{\infty}\Phi^{(2)}_{mn}\cos\lambda_m x+\sum_{m,n=1}^{\infty}\Phi^{(3)}_{mn}\cos\mu_n y$$

$$\Phi^{(1)}_{mn}=-\frac{Eh}{2\left(\lambda_m^2+\mu_n^2\right)^2}C^{(1)}_{mn},\quad \Phi^{(2)}_{mn}=\frac{Eh}{2\lambda_m^4}C^{(2)}_{mn},\quad \Phi^{(3)}_{mn}=-\frac{Eh}{2\mu_n^4}C^{(3)}_{mn} \qquad (2.5)$$

The solution of the homogeneous part of equations (2.1) is presented in the form

$$\Phi^0=\frac{P_x y^2}{2}+\frac{P_y x^2}{2}-2P_{xy}xy \qquad (2.6)$$

Assuming that planar tensions are constant and satisfying the boundary conditions (2. 3), we receive

$$N_x=P_x=\frac{Eh}{8(1-\nu^2)}\sum_{p,q=1}^{\infty}\left(\lambda_p^2+\nu\mu_q^2\right)f_{pq}^2$$

$$N_y=P_y=\frac{Eh}{8(1-\nu^2)}\sum_{p,q=1}^{\infty}\left(\nu\lambda_p^2+\mu_q^2\right)f_{pq}^2,\ T_{xy}=P_{xy}=0 \qquad (2.7)$$

Then the expression for stress function Φ will be

$$\Phi=\frac{P_x y^2}{2}+\frac{P_y x^2}{2}+\sum_{m,n=1}^{\infty}\left[\Phi_{mn}^{(1)}\cos\lambda_m x\cos\mu_n y+\Phi_{mn}^{2}\cos\lambda_m x+\Phi_{mn}^{(3)}\cos\mu_n y\right] \quad (2.8)$$

On the basis of (2. 4) and (2. 8) the nonlinear operator $L(w,\Phi)$ will take the following form

$$L(w,\Phi)=\sum_{m,n=1}^{\infty}F_{mn}\sin\lambda_m x\sin\mu_n y \quad (2.9)$$

where

$$F_{mn}=\sum_{\alpha=m+1,\beta=n+1}^{\infty}F_{\alpha-m,\beta-n}^{(1)}-\sum_{\alpha=m+1,\beta=1}^{\infty}F_{\alpha-m,\beta+n}^{(1)}-\sum_{\alpha=1,\beta=n+1}^{\infty}F_{\alpha+m,\beta-n}^{(1)}+\sum_{\alpha,\beta=1}^{\infty}F_{\alpha+m,\beta+n}^{(1)}+$$

$$+\sum_{\alpha=m+1,\beta=1}^{\infty,n-1}F_{\alpha-m,n-\beta}^{(2)}-\sum_{\alpha,\beta=1}^{\infty,n-1}F_{m+\alpha,n-\beta}^{(2)}+\sum_{\alpha=1,\beta=n+1}^{m-1,\infty}F_{m-\alpha,n+\beta}^{(2)}-\sum_{\alpha,\beta=1}^{m-1,\infty}F_{m-\alpha,n+\beta}^{(2)}+$$

$$+\sum_{\alpha,\beta=1}^{m-1,n-1}F_{m-\alpha,n-\beta}^{(2)}+\sum_{\alpha=m+1}^{\infty}F_{\alpha-m,n}^{(3)}-\sum_{\alpha=1}^{\infty}F_{\alpha+n,n}^{(3)}+\sum_{\alpha=1}^{m-1}F_{m-\alpha,n}^{(3)}+$$

$$+\sum_{\beta=n+1}^{\infty}F_{m,\beta-n}^{(4)}-\sum_{\beta=1}^{\infty}F_{m,\beta+n}^{(4)}+\sum_{\beta=1}^{n-1}F_{m,n-\beta}^{(4)}-P_x\lambda_m^2 f_{mn}-P_y\mu_n^2 f_{mn}$$

$$F_{pq}^{(1)}=\frac{1}{4}\Phi_{pq}^{(1)}\left(\lambda_\alpha\mu_q-\lambda_p\mu_\beta\right)^2 f_{\alpha\beta},F_{pq}^{(2)}=\frac{1}{4}\Phi_{pq}^{(1)}\left(\lambda_\alpha\mu_q+\lambda_p\mu_\beta\right)^2 f_{\alpha\beta}$$

$$F_{pq}^{(3)}=\frac{1}{2}f_{\alpha n}\lambda_p^2\mu_n^2\Phi_{pn}^{(2)},\quad F_{pq}^{(4)}=\frac{1}{2}f_{m\beta}\lambda_p^2\mu_q^2\Phi_{mq}^{(3)}$$

Then we shall receive

$$N_x^0\frac{\partial^2 w}{\partial x^2}+N_y^0\frac{\partial^2 w}{\partial y^2}+2T_{xy}^0\frac{\partial^2 w}{\partial x\partial y}=$$

$$=\left[-\left(N_x^0\lambda_m^2+N_y^0\mu_n^2\right)f_{mn}+T_{xy}^0\sum_{p,q=1}^{0}a_{mpnq}f_{pq}\right]\sin\lambda_m x\sin\mu_n y \quad (2.10)$$

where

$$a_{mpnq}=\frac{32}{ab}\frac{\lambda_m\lambda_p\mu_n\mu_q}{\left(\lambda_m^2-\lambda_p^2\right)\left(\mu_n^2-\mu_q^2\right)},\quad \left.\begin{matrix}m\neq p,\,m+p\\ n\neq q,\,n+q\end{matrix}\right\}=2k+1$$

At last, in accordance with (2. 1), (2. 4), (2. 9) and (2. 10) for unknown f_{mn} we shall receive

$$D\left(\lambda_m^2+\mu_n^2\right)^2 f_{mn}=F_{mn}-\left(N_x^0\lambda_m^2+N_y^0\mu_n^2\right)f_{mn}+T_{xy}^0\sum_{p,q}a_{mpnq}f_{pq}+q_{mn} \quad (2.11)$$

where q_{mn} are coefficients of sine decay of function q(x, y)

It is necessary to have five values of plate deflection in five points for determining unknown $E,E/\left(1-\nu^2\right),N_x^0,N_y^0,T_{xy}^0$. Then we must determine five f_{mn} coefficients from (24). At last we must take five q_{mn} coefficients from the function

$q(x, y)$ decay and we shall determine unknown values from five equations of system (2. 11).

It is necessary to point, that as here as also later on, some of unknown values can be determined from one-dimensional problem.

3. The linear equation of plate free vibrations will be

$$D\Delta^2 w = N_x^0 \frac{\partial^2 w}{\partial x^2} + N_y^0 \frac{\partial^2 w}{\partial y^2} + 2T_{xy}^0 \frac{\partial^2 w}{\partial x \partial y} - \rho h \frac{\partial^2 w}{\partial t^2} \tag{3.1}$$

For simply-supported plate the frequensis are determined from following equations

$$\left[D\left(\lambda_m^2 + \mu_n^2\right)^2 + N_x^0 \lambda_m^2 + N_y^0 \mu_n^2 - \rho h \omega^2\right] f_{mn} - T_{xy}^0 \sum_{p,q} a_{mnpq} f_{pq} = 0 \tag{3.2}$$

If we know four frequencies [4], we must take four equations from (3.2). From the condition that the determinant is equal zero we receive the following system of four equations for determination of N_x^0, N_y^0, T_{xy}^0 and $D\left(or E / \left(1 - \nu^2\right)\right)$

$$\left(\frac{4\pi^2 D}{a^2} + N_x^0 + N_y^0 - \frac{a^2 \rho h}{\pi^2} \omega_{11}^2\right)\left(\frac{64\pi^2 D}{a^2} - 4N_x^0 - N_y^0 - \frac{a^2 \rho h}{\pi^2} \omega_{11}^2\right) = \frac{2^{12}}{3^4}\left(T_{xy}^0\right)^2$$

$$\left(\frac{4\pi^2 D}{a^2} + N_x^0 + N_y^0 - \frac{a^2 \rho h}{\pi^2} \omega_{22}^2\right)\left(\frac{64\pi^2 D}{a^2} - 4N_x^0 - N_y^0 - \frac{a^2 \rho h}{\pi^2} \omega_{22}^2\right) = \frac{2^{12}}{3^4}\left(T_{xy}^0\right)^2$$

$$\left(\frac{25\pi^2 D}{a^2} + N_x^0 + 4N_y^0 - \frac{a^2 \rho h}{\pi^2} \omega_{12}^2\right)^2 = \frac{2^{12}}{3^4}\left(T_{xy}^0\right)^2 \tag{3.3}$$

$$\left(\frac{25\pi^2 D}{a^2} + N_x^0 + 4N_y^0 - \frac{a^2 \rho h}{\pi^2} \omega_{21}^2\right)^2 = \frac{2^{12}}{3^4}\left(T_{xy}^0\right)^2$$

Let us consider an example for illustration. A square plate $(a \times a)$ with thickness h and constant material density ρ is considered. It is known, that $N_x^0 = N_y^0 = 0$ and plate frequencies ω_{11}, ω_{22} are found from an experiment. The bending stiffness D and initial shear tension T_{xy}^0 are determined

$$D = \frac{a^4}{68\pi^4} \rho h\left(\omega_{11}^2 + \omega_{22}^2\right)$$

$$2\left(T_{xy}^0\right)^2 = \frac{a^2 \rho^2 h^2}{(16)^2 (17)^2 \pi^4}\left(\omega_{22}^2 - \omega_{11}^2\right)\left(\omega_{22}^2 - 16\omega_{11}^2\right) \tag{3.4}$$

Here $\omega_{22} \geq 4\omega_{11}$ and $T_{xy}^0 = 0$, when $\omega_{22} = 4\omega_{11}$

4.In general it is impossible to determine the function of nonhomogeneity even in particular case of nonhomogeneous beam by frequences spectrum of a single vibration problem. But there are some problems, when this question has a unique solution. One of these is the problem of beam vibrations with piece-constant along length elasticity moduli

$$E(x) = E_k \text{ when } l_{k-1} \leq x < l_k, \quad k = 1,2,\ldots,n, \quad l_0 = 0, \quad l_n = l \tag{4.1}$$

Such distribution will be possible in a particular case when the beam consists of n anisotropic crystalls and each of the crystalls has different orientation concerning the beam axes.

The direct problem solution, when $E(x)$ is known and material density is constant, is reduced to the solution of following system of equations for simply-supported beam

$$\left[\left(a_0-0{,}5a_{2m}\right)m^4-\Omega^2\right]f_m+\sum_{n=1,m\neq n}^{\infty}0{,}5\left(a_{m-n}-a_{m+n}\right)m^2n^2f_n=0 \qquad (4.2)$$

$$a_0=\frac{1}{l}\sum_{k=1}^{n}E_k\left(l_k-l_{k-1}\right);\quad a_m=\frac{4}{m\pi}\sum_{k=1}^{n}E_k\cos\frac{l_k+l_{k-1}}{2}\frac{m\pi}{l}\sin\frac{l_k-l_{k-1}}{2}\frac{m\pi}{l}$$

From the condition that the determinant of system (4.2) is equal to zero we receive the equation for frequencies ω determining.

It is necessary to have n frequencies (in accordance with unknown E_k number) for inverse problem solving. It is possible to do it by the following way also, to determine E_k in accordance with matrix diagonal members, and late on to put E_k in nondiagonal members as known.

5.It is interesting to consider the inverse problem in the membrane case. The membrane is stretched in the direction of x axis by an unknown tension $T_1(y)$ and in the direction of y axis by an unknown tension $T_2(x)$. The deflection function $u(x,y)$ is known, when normal force $q(x,y)$ is given. The determination of tensions T_1, T_2 is required.

The problem is reduced to solving of equation

$$T_1(y)\frac{\partial^2 u}{\partial x^2}+T_2(x)\frac{\partial^2 u}{\partial y^2}=q(x,y) \qquad (5.1)$$

with boundary conditions

$$x=0,a;\quad u=0$$
$$y=0,b;\quad u=0 \qquad (5.2)$$

Let us consider the direct problem for the particular case at first. It is assumed that the membrane is square and known tensions satisfy the following conditions

$$T_1(y)=T_0f(y),\quad T_2(x)=T_0f(x) \qquad (5.3)$$

It is required to find the deflection function $u(x,y)$ when the force $q(x,y)$ is given.

The direct problem is solved by use of the following presentations

$$u=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\sin\lambda_m x\sin\lambda_n y$$

$$f(x)=\sum_{k=0}^{\infty}a_k\cos\lambda_k x,\quad f(y)=\sum_{k=0}^{\infty}a_k\cos\lambda_k y,\quad a_0=1, \qquad (5.4)$$

$$q=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}q_{mn}\sin\lambda_m x\sin\lambda_n y,\quad \lambda_p=\frac{p\pi}{a}(p=m,n,k)$$

After putting (5.4) in (5.1), collection and making equal to zero coefficients of members $\sin\lambda_m x\sin\lambda_n y$ we receive the following infinite system of equations with respect to unknown constants A_{mn}

$$\lambda_m^2\sum_{k=1}^{n-1}(a_{n-k}-a_{n+k})A_{mk}+\lambda_n^2\sum_{k=1}^{m-1}(a_{m-k}-a_{m+k})A_{kn}+\left[(2a_0-a_{2n})\lambda_m^2+(2a_0-a_{2m})\lambda_n^2\right]A_{mn}+$$

$$+\lambda_m^2\sum_{k=n+1}^{\infty}(a_{k-n}-a_{k+n})A_{mk}+\lambda_n^2\sum_{k=m+1}^{\infty}(a_{k-m}-a_{k+m})A_{kn}=-\frac{2q_{mn}}{T_0} \qquad (5.5)$$

From (5.5) it follows, that coefficients A_{mn} are determined in unique way, when a_k and q_{mn} are known. A_{mn} are determined in unique way independently from conditions (5.3). Hence, the direct problem has unique solution for arbitrary initial tensions. An ordinary method of the direct problem solving is consecutive solving of the shortened system of equations instead of infinite system (5.5). For example, as a first approximation we take, instead of (5.5), the system of four initial equations, in which coefficients A_{11}, A_{12}, A_{21}, A_{22}, are leaved only. Then these coefficients are determined in unique way.

Let us consider the inverse problem for shortened system. There are known A_{11}, A_{12}, A_{21}, A_{22}, (deflection function or, more exactly, four initial coefficients of deflection function desay), q_{11}, q_{21}, q_{12}, q_{22} (normal force function). It is required to determine initial tensions in the case (5.3). From the shortened system we shall have

$$2A_{11}a_2+(A_{12}+A_{21})a_4=2q_{11}\lambda_1^{-2}T_0^{-1}+4A_{11}+A_{12}+A_{21}$$

$$(\lambda_1^2A_{11}+\lambda_2^2A_{22})a_1-\lambda_2^2A_{12}a_2-(\lambda_1^2A_{11}+\lambda_2^2A_{22})a_3-\lambda_1^2A_{12}a_4=$$

$$=-2q_{12}T_0^{-1}-2(\lambda_1^2+\lambda_2^2)A_{12}=-2q_{12}T_0^{-1}-2(\lambda_1^2+\lambda_2^2)A_{12}$$

$$(\lambda_1^2A_{11}+\lambda_2^2A_{22})a_1-\lambda_2^2A_{21}a_2-(\lambda_1^2A_{11}+\lambda_2^2A_{22})a_3-\lambda_1^2A_{21}a_4=$$

$$=-2q_{21}T_0^{-1}-2(\lambda_1^2+\lambda_2^2)A_{21} \qquad (5.6)$$

$$(\lambda_2^2A_{21}+\lambda_1^2A_{12})a_3+2\lambda_2^2A_{22}a_4=2q_{22}T_0^{-1}+\lambda_2^2A_{21}+\lambda_1^2A_{12}+4\lambda_2^2A_{22}$$

The system of four equation (5.6) determines in unique way coefficients a_1, a_2, a_3, a_4.

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